# Week 1 Lecture 2

* Limits of functions §7.8.1
  1. Definition: Given a function f(x) that is defined for for some , we say that the limit of f(x) equals as x goes to , denoted by

If for every >0, there exists a such that

Wherever

* 1. Example:

Show that

* 1. Solution

So fix . Then choose

…

* Properties of limits
  1. Theorem

Suppose f(x) and g(x) are defined on for some and

and

NOTE: Make sure both these limits actually exist.

Then:

1. provided
   1. Theorem (uniqueness of limits)

Limits are unique (If and , then )

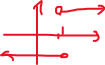
* 1. Non-existence of limits

There can be many reasons why a limit does not exist.

Three common reasons:

1. Jump discontinuities

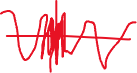
Let



does note exist (DNE)

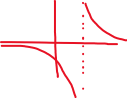
1. Oscillation

Let



1. Unhandedness

Let



* One-sided limits §7.8.2

Definition: Given a function that is defined for for some , we say that the limit of f(x) equals l as x goes to x0 from above, denoted by:

,

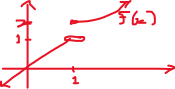
If for every there exists a such that

Wherever .

We can define the limit of f(x) as x goes to x0 from below in a similar fashion, if it exits, is denoted by:

* 1. Example

Let



Then

But

* 1. Theorem

if and only if

* Continuous functions §7.9

Definition: Given a function f(x) that is defined for for some , we say that f(x) is continuous at if

(i.e. limit exists AND agrees with function)

* 1. Examples: The following functions are continuous everywhere they are defined:
     + Polynomials and rational functions
     + Exponential and logarithmic functions
     + Trigonometric functions
     + Compositions of continuous functions
* Properties of continuous functions §7.9.1
* Theorem: Suppose f(x) is continuous at [a,b]. Then
  1. There exists points such that
  2. For any and there exists such that . (Intermediate Value Theorem).

